ENGINEERING METHOD FOR CALCULATING HEAT-CONDUCTION PROCESSES

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A method for averaging the time derivative of the temperature in the heat-conduction equation is discussed and used to calculate the heating of a plate.

For a number of problems of practical interest it is expedient to use the hypothesis of the finite velocity of the temperature front [1]. Thus, for example, certain products of animal origin (biological media) have a specific combination of thermophysical and structural-mechanical properties necessitating taking account of the finite velocity of the temperature front.

According to Lykov [1] the velocity of propagation of heat is given by the expression

$$w_r = \lambda / c\gamma\tau_r. \tag{1}$$

The quantity τ_r can, to a certain extent, be regarded as a characteristic of the structural-mechanical properties of the body.

According to [2, 3] the period of relaxation for meat products depends on a complex of structural-mechanical properties and can be estimated to be of the order of 20-30 sec $\simeq$ 0.00675 h on the average.

Using this estimate of the period of relaxation and taking $\lambda = 0.4$ kcal/m · h · deg, $c = 0.8$ kcal/kg · deg, $\gamma = 1000$ kg/m^3 [5] we find from Eq. (1) the order of magnitude of the velocity of propagation of heat in meat products:

$$w_r^2 = 0.0073 \ \text{m}^2/\text{h}^2,$$

from which

$$w_r = 0.0855 \ \text{m/h}.$$

Thus the velocity of propagation of heat in meat products is of the order of magnitude of 10^{-1} m/h, which in our opinion completely justifies the hypothesis of a temperature front.

We now consider the problem of the heating of an infinite plate with constant thermophysical coefficients, introducing the hypothesis of the finite velocity of the temperature front. This problem can be formulated rigorously in the following form:

$$\frac{\partial U}{\partial \text{Fo}} = \frac{\partial^2 U}{\partial \xi^2}, \tag{1a}$$

$$U(\xi, 0) = 0, \tag{1b}$$

$$\left[\frac{\partial U}{\partial \xi}\right]_{\xi=0} = 0, \tag{1c}$$

$$\left[\frac{\partial U}{\partial \xi} + \text{Bi}\, U\right]_{\xi=1} = \text{Bi}, \tag{1d}$$

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where

$$U = \frac{u - u_0}{u_1 - u_0}; \quad \xi = x/L; \quad \mathrm{Fo} = \frac{at}{L^2}; \quad \mathrm{Bi} = \frac{\alpha}{\lambda} L.$$

The problem is to find the solution of Eq. (1a) which satisfies conditions (1b) and (1c).

In addition to the assumption of a temperature front propagating with a finite velocity we assume that: 1) the temperature front in the plate moves symmetrically with respect to the median plane; 2) a moving boundary of the thermal perturbation exists such that all points lying outside this boundary have a temperature different from the initial value, and all points on the boundary have the initial temperature; 3) $\partial U/\partial\xi = 0$ at $\xi = \zeta$, and $\zeta = \zeta(\mathrm{Fo})$ is the coordinate of the boundary of propagation of the thermal perturbation region.

Assumptions 1)-3) essentially imply that the propagation of heat is divided into two phases.

The first phase covers the period during which the temperature front propagates from some boundary plane to the median plane.

The second phase begins at the instant the temperature front reaches the median plane.

Henceforth in considering the temperature distribution in a plate we average the derivative with respect to Fo and replace problem (1) by the following approximate problem:

$$\frac{\partial^2 U^{(1)}}{\partial \xi^2} = 2\varphi^{(1)}(\mathrm{Fo}), \tag{2a}$$

$$[U^{(1)}(\xi;\ \mathrm{Fo})]_{\xi=\zeta} = 0, \tag{2b}$$

$$\left[\frac{\partial U^{(1)}}{\partial \xi}\right]_{\xi=\zeta} = 0, \tag{2c}$$

$$\left[\frac{\partial U^{(1)}}{\partial \xi} + \mathrm{Bi}\, U^{(1)}\right]_{\xi=1} = \mathrm{Bi}, \tag{2d}$$

$$2\varphi^{(1)}(\mathrm{Fo}) = \frac{1}{1-\zeta}\int_{\zeta}^{1} \frac{\partial U^{(1)}}{\partial \mathrm{Fo}}\, d\xi. \tag{2e}$$

Here $U^{(1)}$ is the temperature for calculating the first phase. Integrating (2a) and satisfying (2b)-(2d) we find an expression for the temperature distribution in the plate:

$$U^{(1)} = \mathrm{Bi}\, \frac{(\xi-\zeta)^2}{\mathrm{Bi}(1-\zeta)^2 + 2(1-\zeta)}. \tag{3}$$

The function $\zeta = \zeta(\mathrm{Fo})$ is determined from condition (2e), assuming $\zeta = 1$ for $\mathrm{Fo} = 0$

$$\mathrm{Fo} = \frac{(1-\zeta)^2}{12} + \frac{1-\zeta}{3\,\mathrm{Bi}} - \frac{2}{3\,\mathrm{Bi}^2}\ln[1 + 0{,}5\,\mathrm{Bi}(1-\zeta)]. \tag{4}$$

The duration of the first phase $\mathrm{Fo}^{(1)}$ is determined from Eq. (4) by setting $\zeta = 0$.

The temperature distribution during the second phase is calculated in the following way. We consider an approximate problem similar to (2):

$$\frac{\partial^2 U^{(2)}}{\partial \xi^2} = 2\varphi^{(2)}(\mathrm{Fo}), \tag{5a}$$

$$U^{(2)}(0;\ \mathrm{Fo}^{(1)}) = 0, \tag{5b}$$

$$\left[\frac{\partial U^{(2)}}{\partial \xi}\right]_{\xi=0} = 0, \tag{5c}$$

$$\left[\frac{\partial U^{(2)}}{\partial \xi} + \mathrm{Bi}\, U^{(2)}\right]_{\xi=1} = \mathrm{Bi}, \tag{5d}$$

$$2\varphi^{(2)}(\mathrm{Fo}) = \int_{0}^{1} \frac{\partial U^{(2)}}{\partial \mathrm{Fo}}\, d\xi. \tag{5e}$$

The solution of problem (5) has the form

$$U^{(2)}(\xi,\mathrm{Fo}) = 1 - \frac{\mathrm{Bi}}{\mathrm{Bi}+2}\left(\frac{\mathrm{Bi}+2}{\mathrm{Bi}} - \xi^2\right)\exp\left[-\frac{3\,\mathrm{Bi}}{\mathrm{Bi}+3}\left(\mathrm{Fo} - \mathrm{Fo}^{(1)}\right)\right]. \tag{6}$$

A comparison of our result with the classical solution of A. V. Lykov [1] shows practically complete agreement for $\mathrm{Fo} > 2\mathrm{Fo}^{(1)}$. For $\mathrm{Fo} < 2\mathrm{Fo}^{(1)}$, $\mathrm{Bi} = 10$, and $\xi = 0$ the classical result is larger than that found from Eqs. (3), (4), and (6) by 7.5-8%. The difference increases with either decreasing or increasing Bi.

Our result can be extended to the calculation of temperature distributions in spheres and infinitely long circular cylinders.

NOTATION

w_r	is the velocity of propagation of heat, m/h;
c	is the specific heat, J/kg · deg;
γ	is the density of the medium, kg/m^3;
τ_r	is the period of relaxation of elastic shear stress, h;
u	is the temperature of plate, deg;
u_1	is the ambient temperature, deg;
x	is the running coordinate, m;
L	is the characteristic dimension (half-thickness of plate), m;
α	is the heat-transfer coefficient, W/m^2·deg;
λ	is the thermal conductivity, W/m·deg;
a	is the thermal diffusivity, m^2/h;
$U = (u - u_0)/(u_1 - u_0)$	is the dimensionless temperature;
$\xi = x/L$	is the dimensionless coordinate;
$\mathrm{Fo} = at/L^2$	is the Fourier number;
$\mathrm{Bi} = \alpha L/\lambda$	is the Biot number;
$U^{(1)}, U^{(2)}$	are the dimensionless temperatures during first and second phases;
$\xi = \zeta(\mathrm{Fo})$	is the equation of thermal perturbation front;
$\mathrm{Fo}^{(1)}$	is the value of Fo at end of first phase.

LITERATURE CITED

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